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Search for multifractal features in cherenkov arrival time

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Extensive air shower products are fractal in nature. Both simulated and experimental Cherenkov images display multifractal properties. In this paper we explore the possibility of searching multifractal features in cherenkov arrival times.

1. Motivation

Extensive air shower (EAS) is a multifractal process because of multiplicative nature of bremsstrahlung and pair production. It has been shown that EAS products like Cherenkov photons, electron density distribution etc are multifractal in nature [2,3]. In this paper we search for multifractal features in the temporal character of EAS.

2. Multifractal formalism

Fractals are self-similar objects which look same on many different scales of observations. Fractals are defined in terms of Hausdroff-Bescovitch dimensions. Fractal dimensions characterize the geometric support of a structure but can not provide any information about a possible distribution or a probability that may be part of a given structure. This problem has been solved by defining an infinite set of dimensions known as generalized dimensions which are achieved by dividing the object under study into pieces ,each piece is labelled by an index i=0,1,2....N. If we associate a probability p_i with each piece of size l_i than partition function is obtained which permits to define generalized dimension D_q

$$(q-1)D_q = \tau(q) \tag{1}$$

Here q is a parameter which can take all values between -∞ to ∞. $\tau(q)$ is obtained from scaling properties of the partition function. This formalism is called as multi-fractal formalism which characterizes both the geometry of a given structure and the probability measure associated with it. There are infinite set of other exponents from which information can be obtained by constructing an equivalent picture of the system in terms of scaling indices 'α' for the probability measure defined on a support of fractal dimension f(α). This is achieved by defining probability measure p_i in terms of α. α(q)) is the fractal dimension of the set.

3. Approach followed in the present paper

In this approach suggested by Chhabra et al [3] whole experimental /simulation measure is covered with boxes of size l and probability $P_i(l)$ is computed. From this probability construct a one parameter family of normalized measure $\mu(q)$

$$\mu_i(q,l) = \frac{[P_i(l)]^q}{\sum_j [P_j(l)]^q} \tag{2}$$

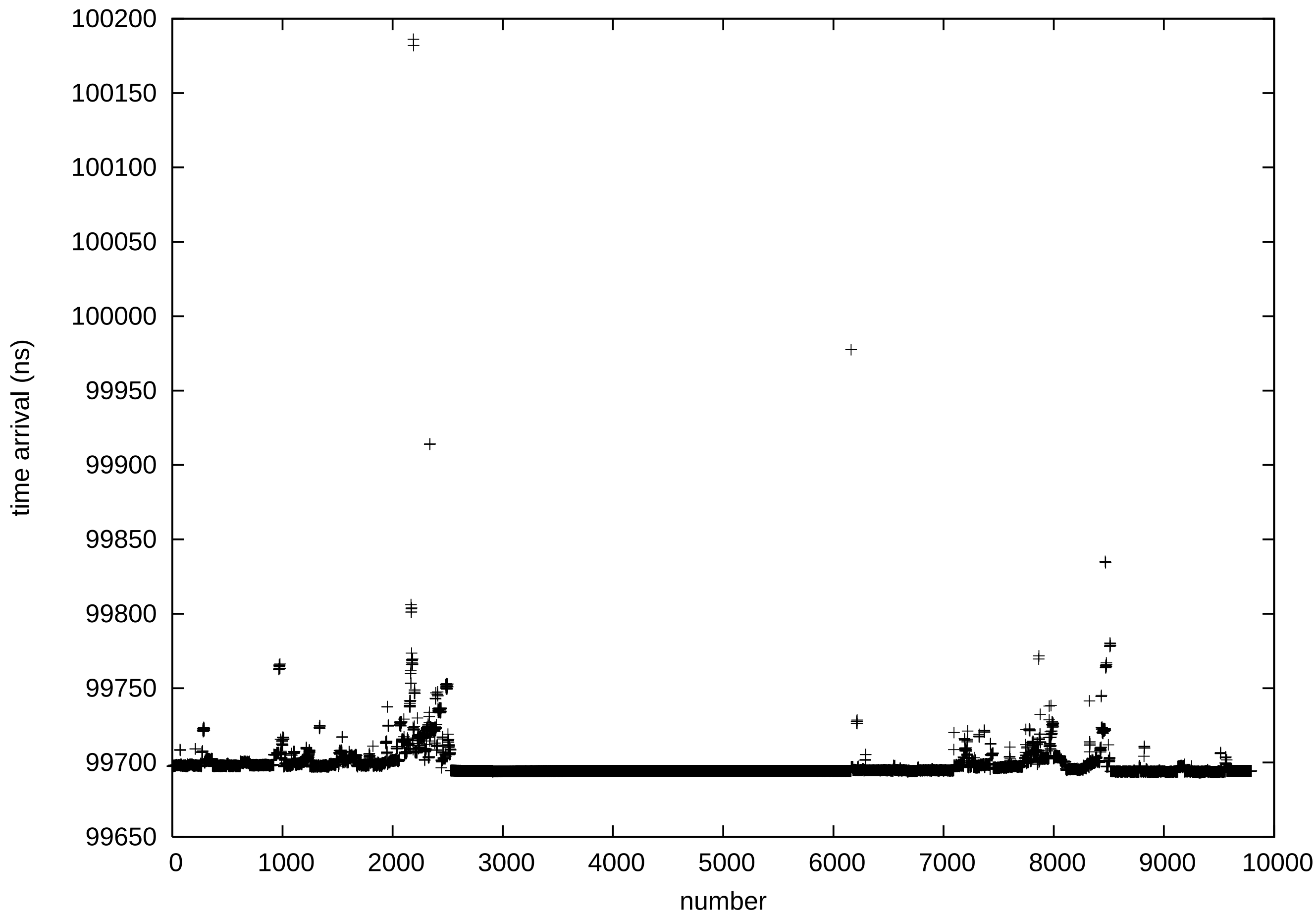

Figure 1. Cherenkov arrival time of a 2 TeV proton initiated shower.

The Hausdroff dimension of this measure is given as

$$f(q) = \lim_{l \to 0} \frac{\sum_i (q,l) ln \mu_i(q,l)}{lnl} \tag{3}$$

The corresponding singularity strength α is given as

$$\alpha(q) = \lim_{l \to 0} \frac{\sum_i (q,l) ln P_i(q,l)}{lnl} \tag{4}$$

As in the case of generalized dimensions, parameter q here also works as a microscope to explore different regions of singular measure [3]. For q $>$1, $\mu(q)$ amplifies the singular regions of the measure. For q $<$, $\mu(q)$ accentuates the lesser regions of singular measure. For q=1, the measure $\mu(1)$ replicates the original measure.

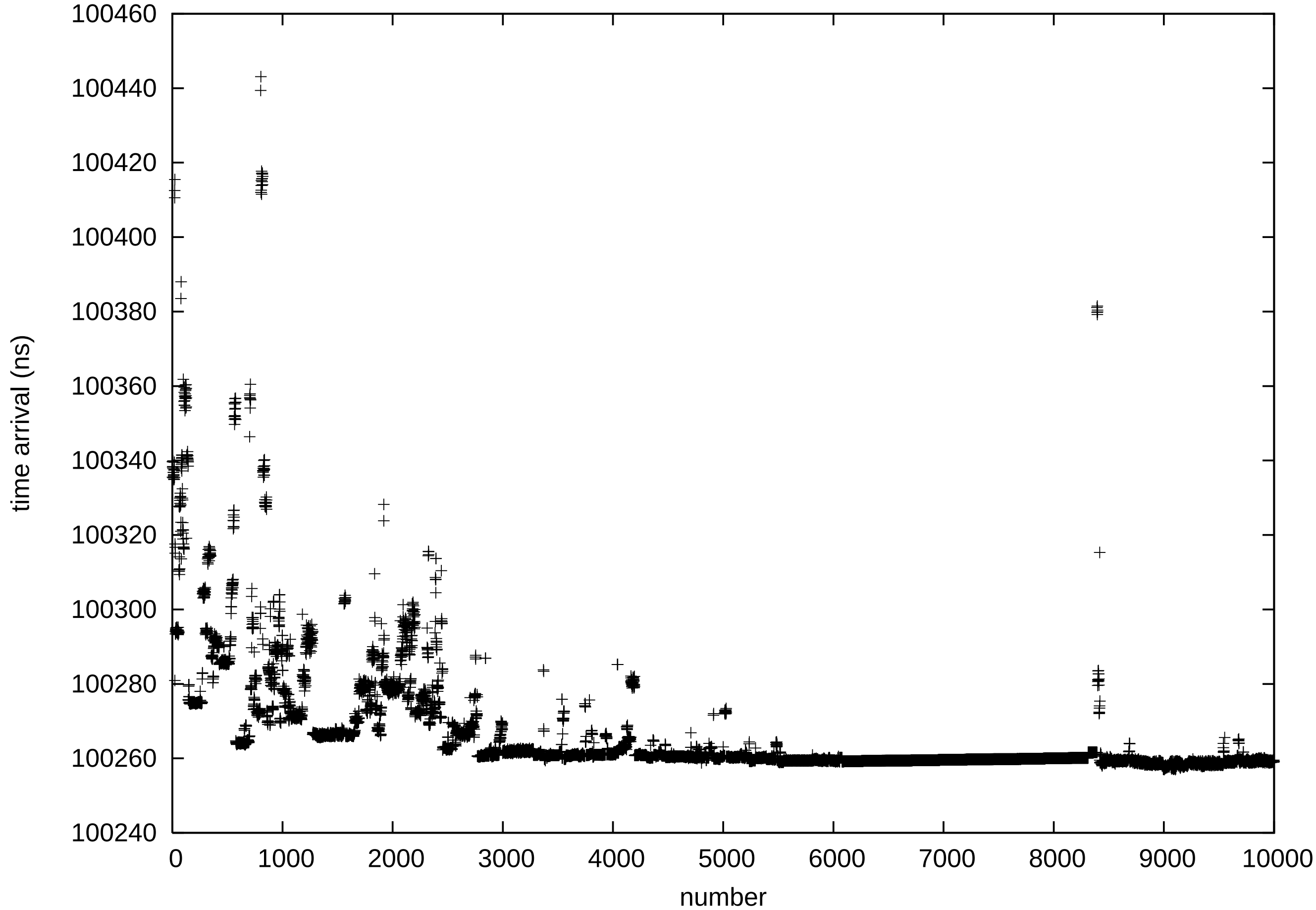

Figure 2. Cherenkov arrival time of 1 TeV γ-ray initiated shower.

4. Simulation studies

In the present studies simulations were carried out using CORSIKA (version 5.6211) along with EGS4, VENUS,GHEISHA codes for Cherenkov option. Simulated data is generated for TACTIC [2] like configuration, each element of the size 4m X 4m. Simulated data corresponds to Mt.Abu altitude (1300 m) and appropriate magnetic field. Cherenkov arrival time data corresponds to wavelength band of 300-450 nm. Cherenkov arrival time data for γ-rays and protons were generated for energies of 1 TeV and 2 TeV respectively. It is observed that there are lot of fluctuations from shower to shower basis. To do multifractal analysis we choose only that γ-ray and proton showers where number of photons generated were roughly same. In cherenkov arrival times of γ-rays and protons probability $P_i(l)$ was calculated for various box sizes. By calculating $\mu_i(q, l)$ in each box Hausdroff dimension and singularity strength was computed. It is clear from figure 3 that for both γ-ray and proton initiated showers we obtain multifractal spectrum. f(α(q)) is the fractal dimension of the set.

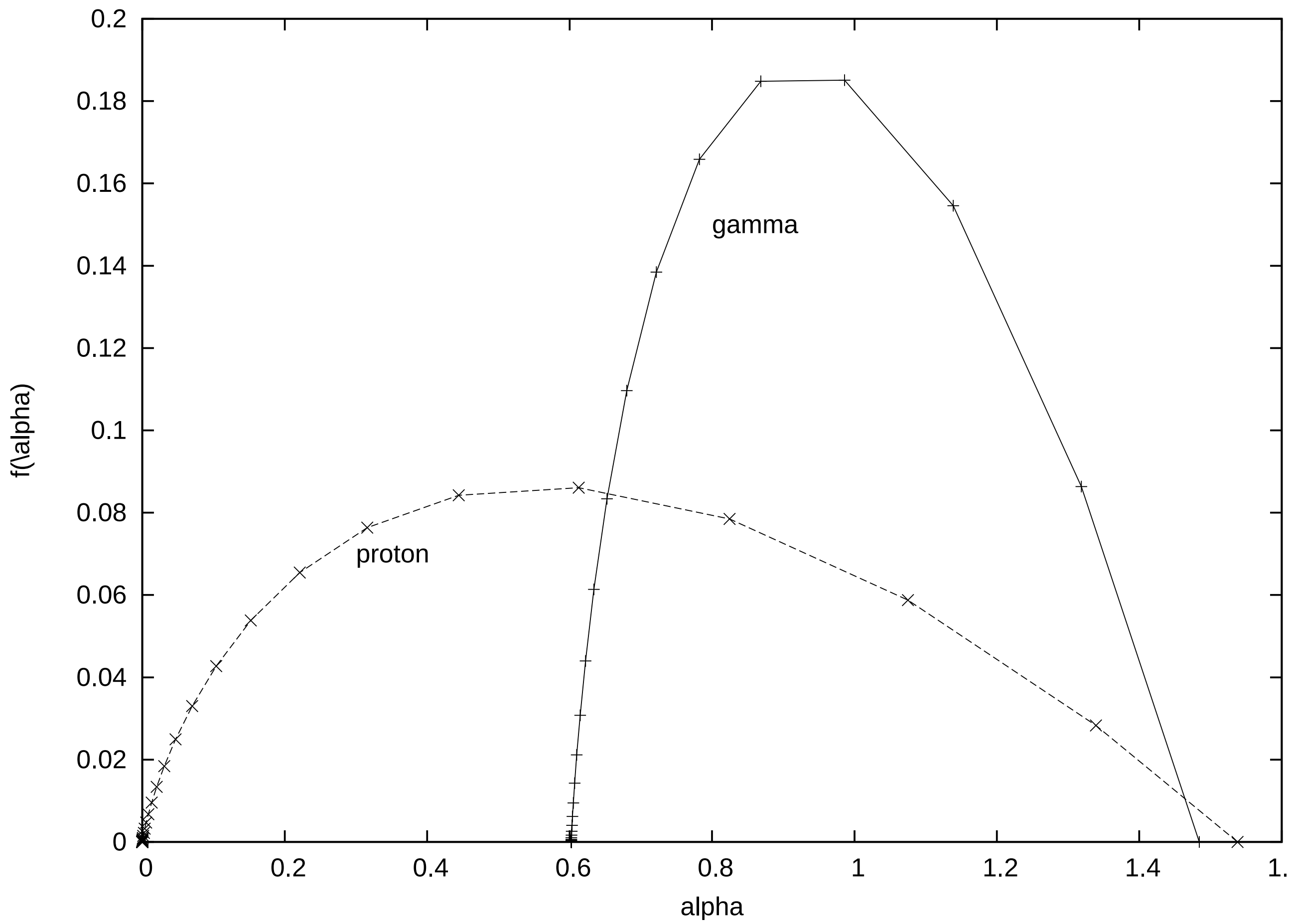

Figure 3. Multifractal spectrum of cherenkov arrival times

5. Conclusion

In this paper it is shown that temporal character of EAS is multifractal in nature. Earlier we had shown that spatial character of EAS is multifractal [1,2]. So it can be concluded that EAS is a multifractal process both in spatial and temporal manifestation.

References

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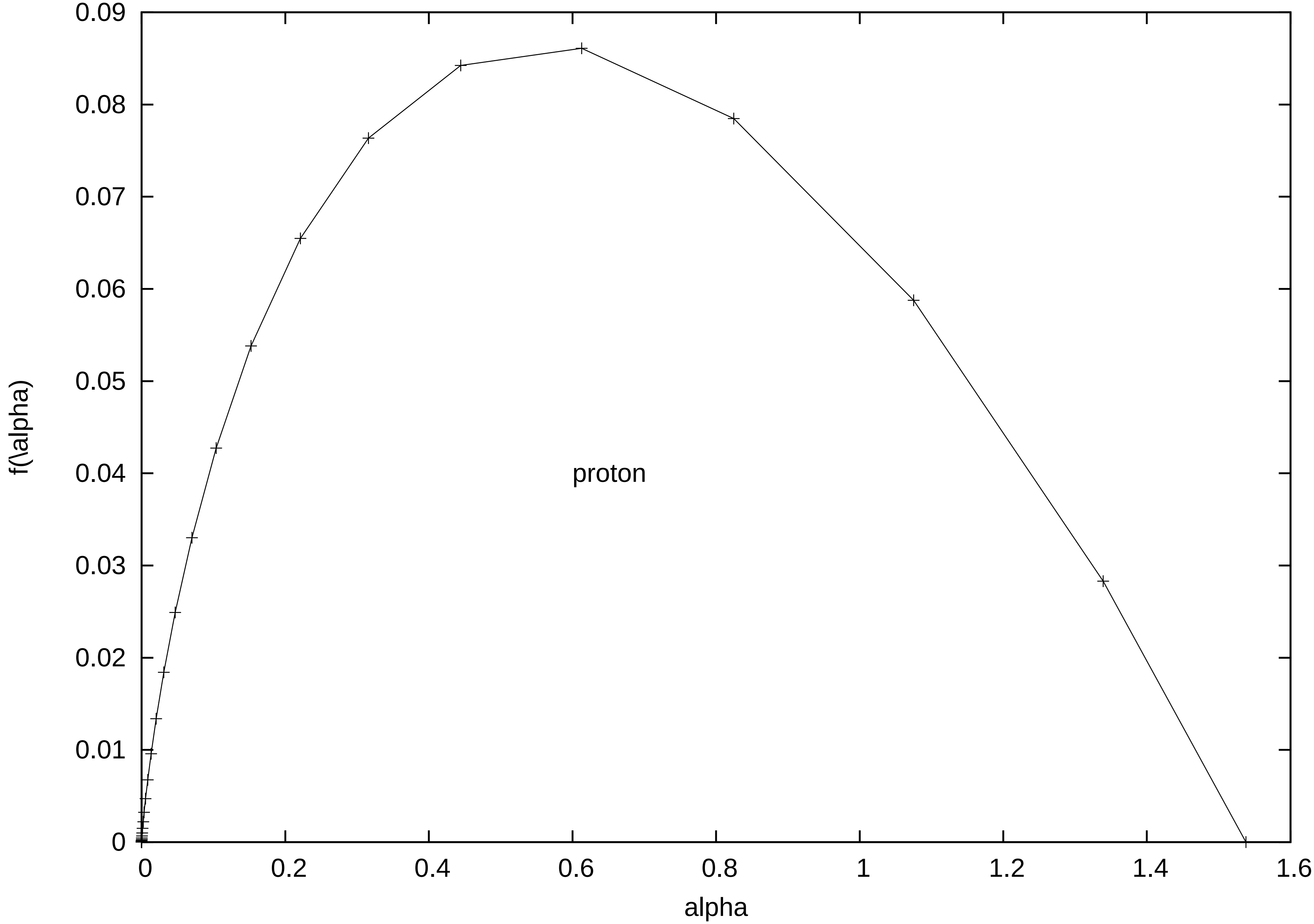

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0.08
0.07
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f(\alpha)
proton
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1
1.2
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alpha